

On the Nature of Black Hole Entropy

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Abstract. I argue that black hole entropy counts only those states of a black hole that can influence the outside, and attempt (with only partial success) to defend this claim against various objections, all but one coming from string theory. Implications for the nature of the Bekenstein bound are discussed, and in particular the case for a holographic principle is challenged. Finally, a generalization of black hole thermodynamics to “partial event horizons” in general spacetimes without black holes is proposed.

BLACK HOLE ENTROPY AND INTERNAL STATES

Let me begin by giving several reasons why we should not think that the Bekenstein-Hawking entropy $S_{\text{BH}} = A/4\hbar G$ of a black hole counts the number of internal states of the black hole. (By “the entropy” of a black hole I will always mean $A/4\hbar G$ in this article.) These reasons have been enunciated in a thoughtful article by Rafael Sorkin [1], which I will borrow from here.

1. The spatial region inside a black hole horizon can have arbitrarily large volume, with room for an arbitrarily large number of states. For example a Friedmann universe of any size can be joined to the interior of a Schwarzschild black hole. Thus the number of possible internal states of a black hole is unbounded.
2. A black hole is not in “internal equilibrium”, so why should its thermodynamic entropy refer to its interior states?
3. Conditions inside the horizon are causally disconnected from the outside, so how can the states inside be thermodynamically relevant to the outside?
4. According to local quantum field theory the evaporation of a black hole is unitary, at least until the final stages, and the Hawking radiation is correlated to field degrees of freedom inside the black hole. The number of internal states of the black hole must therefore remain large enough to store all the correlations maintaining the purity of the total state. As a black hole evaporates, however, its area and therefore its entropy decreases. Thus the entropy must not be counting the number of internal states.

Regarding point 1, it should be mentioned that the example given will have a white hole horizon and singularity in its past (assuming the weak energy condition holds) so it is not a configuration that would evolve from an ordinary collapse process [2]. It is nevertheless a possible state of the black hole.

There is by now a “standard” argument against points 3 and 4, namely, that local quantum field theory may be inapplicable. This argument is suggested by (but not restricted to) string theory, in which local quantum field theory is only an approximation valid under certain conditions. It has been argued both on general principles [3] and in string theory [4] that there are no truly local observables in quantum gravity and that for this reason the decomposition of the Hilbert space into sectors inside and outside the black hole is invalid from the beginning. While this may indeed be true at some fundamental level, the relevant question here is whether local quantum field theory holds to a sufficiently good approximation for points 3 and 4 to be valid. Since the black hole can be macroscopic and the curvature can be very small compared with the string length or Planck length, it is hard for me to see why the local field theory approximation should fail in this regard. To postulate such a mysterious failure, when simpler scenarios exist, seems to me uncalled for radicalism, although it is a hypothesis favored by many physicists today.

BLACK HOLE ENTROPY AND SURFACE STATES

The previous arguments point to the conclusion that black hole entropy is a measure of only those states that can influence the outside of the black hole.¹ These states must be associated with the presence of the horizon, otherwise they would simply be counted as ordinary states of the exterior itself.

One interpretation of this “surface entropy” is that it measures the information in the entanglement of the vacuum across the horizon (“entanglement entropy”) [7]. For fields on a fixed background this is equivalent [8] to the entropy of the thermal state (“thermal atmosphere”) that results when the state is restricted to the outside [9]. This entropy diverges, but gives something of the correct order of magnitude if a Planck scale cutoff is imposed.

It is insufficient to consider fields on a fixed background however. For one thing, although the contributions of quantum fields can be thought of as “loop corrections” to the black hole entropy, there is also a classical contribution coming from the gravitational action itself. One can imagine an induced gravity scenario [10–13], in which the entire gravitational action is induced by matter, however there is still

¹⁾ The case for a surface interpretation of black hole entropy has been made by various authors. In particular, an article by Banks [5] (written before the age of D-branes) makes the case with many of the same arguments as used here, and the argument that the universality of black hole entropy (in spite of the *non-universal* history of the black hole) arises from the universality of the near-horizon geometry was made in a paper by Parentani and Piran [6].

another problem: for non-minimally coupled scalar fields or gauge fields, the entanglement entropy is not equal to the corresponding contribution to the entropy computed from the induced gravitational action. It seems that the difference between these two entropies can be understood as a consequence of the fact that the background itself varies when the temperature is varied [14,15]. Physically, this means that to understand the entropy one must count states in the coupled matter-gravity vacuum.

The large and universal number of states per unit of surface area seems to be explained by the infinite redshift at the horizon: many states at short distances near the horizon have the same, low, energy. In fact, the number would appear to be *infinite* from perturbative counting, but the final count requires knowledge of only the low energy effective gravitational action and the associated low energy Newton constant, as long as the spacetime curvature is small compared with Planck curvature [16,11,17]. Although we are unable to compute the renormalized Newton constant from quantum gravity, its (finite) value can be measured and used in the entropy formula.

Entanglement entropy and the generalized second law

Sorkin proposed a derivation of the generalized second law based on the entanglement interpretation of black hole entropy [1,18]. His idea was that the total entropy $S_{\text{outside}} = S_{\text{horizon}} + S_{\text{rest}}$ of the reduced density matrix outside the horizon receives a large universal contribution S_{horizon} from the vicinity of the horizon and the rest S_{rest} is primarily just the ordinary entropy of a mixed state outside. Invoking the dynamical autonomy of the evolution outside the horizon, Sorkin argues that S_{outside} cannot decrease, which amounts to the usual generalized second law provided S_{horizon} can be identified with the black hole entropy. This explanation of the generalized second law seems so natural that it is hard to believe there is not some truth in it. Unfortunately, as mentioned above, the entanglement interpretation of black hole entropy does not seem to work, but perhaps this conclusion is premature. Perhaps the black hole entropy could yet be understood in terms of entanglement entropy if, as proposed in [19], the division of the system into inside and outside is referred to an intrinsic feature of the fluctuating geometry such as the minimal throat area on some preferred spacelike slice.

OBJECTIONS

Objections can be raised to the assertion that black holes have many more states than are counted by the black hole entropy. I believe that all of these objections are wrong, but it is challenging and instructive to try to point to exactly where they are wrong. I will try to do so here with regard to several objections, all but the first coming from string theory.

Black hole pair creation amplitudes

Semiclassical calculations of black hole pair creation rates display a factor $\exp S_{\text{BH}}$ which admits the natural interpretation as a density of states factor [24]. This seems to lend solid support to the interpretation of $\exp S_{\text{BH}}$ as the number of states of the black hole. If the black hole had more states, would they not contribute to the pair creation rate? This question has been discussed in the past, with conflicting conclusions [20,21,5,22,23], and it deserves to be discussed further. Here I will only state the reason for my belief that the answer is no²: Pair creation is an exponentially suppressed tunneling process, and any “unnecessary” decoration of the black holes would, it seems, be even more suppressed. All the extra internal states are unnecessary decoration, and are therefore essentially irrelevant to the pair creation rates.

String theory

Calculations of black hole entropy in string theory and its descendants have been carried out in several contexts yielding agreement with the Bekenstein-Hawking entropy. In all cases it appears that one is indeed counting all of the states of the object identified with a black hole. Can this be compatible with the claim that black hole entropy does *not* count all of the states? I will attempt to argue that it can, pointing to where one might find the other states. My attempts are only partly successful, and are particularly weak in the context of the AdS/CFT duality.

D-branes

The entropy of certain near-extremal configurations of D-branes has been found to agree with the semiclassical entropy of the black hole configurations with the same set of charges (see for example [25,26]). In the extremal case, for the supersymmetric BPS states, this is understood as a consequence of the fact that the D-brane configuration evolves into the black hole as the string coupling is increased from weak to strong, all the while maintaining the supersymmetry. The enumeration of BPS states is independent of the coupling, hence the agreement in the count of states. In the D-brane picture there is nothing corresponding to the inside of the black hole where extra states can reside so, given the agreement with the black hole entropy, how could a black hole have any more states? For the BPS states the answer is simple: the black hole also does not have any interior in the sense that on a spacelike slice orthogonal to the timelike Killing field the horizon is infinitely far away and has no other side.

The D-brane and black hole entropies also agree for near-extremal states however. In these cases, one can not give such a simple answer. Imagine for instance a

²⁾ I was asked this question during my talk and had no quick answer. After the talk Renaud Parentani suggested the following answer.

configuration that has been maintained at fixed energy above extremality for a long time with the help of an influx of energy equal in magnitude to the Hawking flux. In the black hole picture there is an arbitrarily large amount of information stored in the correlations between the inside and outside of the black hole, so there must be a correspondingly large number of states for the interior. In the D-brane description however there is nothing that corresponds to the interior. How could there be such a drastic mismatch between the total number of states in the two descriptions and still be such agreement on not only the entropy but also the rate of Hawking emission (i.e. the “greybody factors”)?

I can give no really satisfying answer to this question. Surely one has less control over the correspondence between strong and weak coupling away from the BPS sector. It is conceivable that the initial rates for Hawking radiation agree but the details about the correlations that develop over time do not match. In this scenario, there would simply be more non-BPS states at strong coupling than there are at weak coupling. This is not so hard to imagine, since in going from weak to strong coupling the causal structure of the background spacetime is distorted into that of a black hole. An analogy that may be useful is the coupling constant dependence of the state space of electrons in an atom. At sufficiently strong electric coupling the ground state becomes unstable and electrons can be absorbed into the nucleus, at which point the nuclear Hilbert space comes into play in resolving the physics. A strength of this analogy is that in the black hole case the ergoregion inside the (non-extremal) horizon also manifests a kind of instability of the ground state.

AdS/CFT duality

The near-horizon limit of the D-brane physics led to the celebrated Maldacena conjecture, according to which supergravity/string theory in an asymptotically Anti-deSitter spacetime is equivalent to a superconformal field theory on the conformal boundary of that spacetime [27]. An example of this is the duality between superstring theory on $AdS_5 \times S^5$ and a $U(N)$ super-Yang-Mills theory on $S^3 \times R$, where N is related to the string coupling g_s the string length ℓ_s and the AdS radius R by $R^4 = 4\pi g_s N \ell_s^4$, and the Yang-Mills and string couplings are related by $g_{YM}^2 = 4\pi g_s$. There is much remarkable evidence in favor of the AdS/CFT duality, and no evidence against it to date. Hence, for the sake of argument, let us suppose it is valid and ask about the consequences for black holes.

In the $AdS_5 \times S^5$ example it has been shown that the entropy of a black hole which is large compared to the AdS radius (and is hence stable) is $3/4$ of the entropy of a thermal state in the Yang-Mills theory at weak 't Hooft coupling ($g_{YM}^2 N \ll 1$) at the corresponding Hawking temperature. Moreover, there is reason to believe that the entropy would only change by a factor of order unity if the calculation could be done at strong 't Hooft coupling (which is what is required by for the case of large AdS radius).

We thus have a puzzle similar to that in the case of the D-brane state counting,

but now far from extremality. In the Yang-Mills theory it seems there can be no missing states corresponding to the degrees of freedom inside of the black hole. The entropy of the thermal state simply counts all states so, if the Maldacena conjecture is really true, one infers that there can be no independent degrees of freedom inside the black hole. Can this conclusion be evaded?

A simple evasion is to suppose that the equivalence conjectured by Maldacena actually relates the supergravity observables only *outside* the horizon to the Yang-Mills observables in the boundary theory (see Fig. 1(a)). This would be consistent

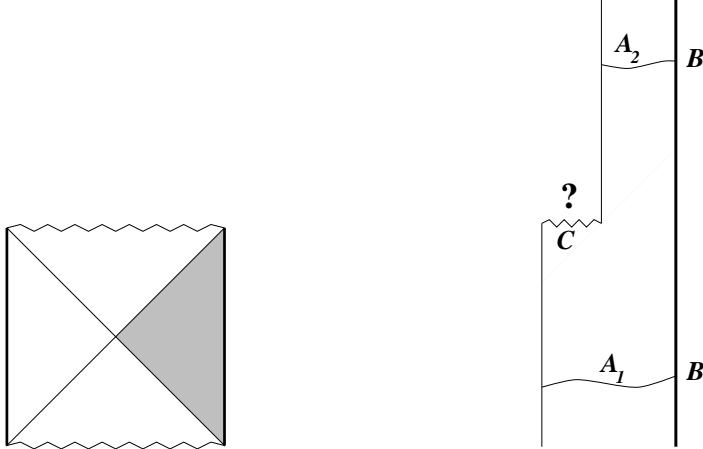


FIGURE 1. (a) Schwarzschild-AdS spacetime. The conformal boundary is the pair of thick vertical lines. A single copy of the Yang-Mills theory on the boundary may just correspond to the gravity theory in the shaded wedge. (b) Black hole formation and evaporation in Anti-de Sitter spacetime. The algebra of observables \mathcal{A}_2 may be a proper subalgebra of \mathcal{A}_2 .

with causality and would certainly explain why all the states inside the black hole are not seen in the Yang-Mills theory. In fact, something like this seems almost necessary in view of the fact that the full Schwarzschild-AdS spacetime has a boundary with two disconnected pieces, the dark vertical lines in Fig. 1(a). What would be the role for the states in the Yang-Mills theory on the left if the one on the right already covered all the states inside the black hole?

A different evasion is required if we consider not an eternal black hole but rather a black hole, small compared with the AdS radius, that forms from collapse and then evaporates (see Fig. 1(b)). In this case the AdS/CFT duality presumably states that *all* observables in the spacetime have correspondents in the CFT on the single boundary component. In particular, observables in the algebra C localized behind the horizon (which is of course only defined relative to a particular state $|\psi\rangle$ of the CFT which corresponds to matter collapsing to form a black hole) must be contained within the full algebra B of observables in the CFT. This in itself is not mysterious, since the field equations allow us to express any observable in C as an observable in the algebra A_1 localized at a spacelike slice before the black hole ever formed.

The question of whether there are independent states of the black hole interior is perhaps most sharply formulated here as the question whether the black hole evaporation is unitary from the viewpoint of the exterior [28]. Since the CFT itself is unitary, the question amounts to whether the algebra A_2 of observables on a spacelike slice after the black hole has evaporated completely is equal to A_1 or is rather a proper subalgebra of A_1 . In the latter case one would need also the observables in the algebra C behind the horizon to fill out the complete algebra. Moreover, causality would suggest that C and A_2 would commute relative to the state $|\psi\rangle$, that is, expectation values of the ideal generated by the commutator algebra $[C, A_2]$ would vanish.

Many practitioners of duality have argued that the equality $A_2 = A_1$ is assured because the situations before collapse and after evaporation are similar: Anti-de Sitter spacetime with some matter. In particular, the “initial” configuration could have been the result of a prior black hole formation and evaporation, or of many cycles of formation and evaporation. If each black hole has internal observables not captured on the outside after the black hole is gone, then one seems to be requiring that the CFT contains within it (relative for an appropriate state) commuting subalgebras of observables corresponding to an infinite number of such black hole interiors, all of which commute with an algebra of outside observables such as A_2 . This requirement seems difficult to reconcile with reasonable expectations about the number of states in the CFT at a given energy.

Can one really arrange a sequence of black hole formations and evaporation where each black hole is made from the Hawking radiation into which the previous black hole evaporated? If not, then a state which produces many black holes must contain to begin with energy corresponding to each black hole. In this case there are perhaps more states so let us suppose, to be difficult, that one can indeed repeatedly refocus the Hawking radiation to form an endless cycle of black holes with a finite amount of energy. In this case either the collection of commuting interior subalgebras exists, or one must deny the independence of the interior observables. Most string theorists support the second alternative. I prefer the first since it requires only nonintuitive behavior of the unfamiliar strongly coupled, (astronomically) large N gauge theory, rather than gross violations of locality where they would not otherwise be expected.

Matrix Theory

Matrix theory (a candidate for a nonperturbative formulation of string theory) can purportedly describe formation and evaporation of black holes, and the theory is manifestly unitary. There seems to be no room in matrix theory for any states corresponding to the interior of a black hole, left over after all particles in the Hawking radiation have dissipated [29]. I have not yet learned enough about matrix theory to think carefully about whether or not there is any loophole through which this conclusion can be evaded.

NATURE OF THE BEKENSTEIN BOUND

The “Bekenstein bound” [30] on the entropy that can be associated with a closed 2-surface Σ is

$$S_\Sigma \leq \frac{A_\Sigma}{4\hbar G}. \quad (1)$$

This is (presently) a heuristic notion motivated by the generalized second law of thermodynamics as follows. Suppose that by tossing in a suitable arrangement of matter the surface Σ could be made to coincide with a slice of the horizon of a black hole. Then the entropy of that black hole would be $A_\Sigma/4\hbar G$, which would violate the second law unless the entropy S_Σ associated with Σ if the extra matter is *not* tossed in is *less*, i.e. unless the bound (1) holds.

In describing the Bekenstein bound I was careful to refer to S_Σ as the entropy *associated with* Σ , rather than the entropy *contained within* Σ , since the meaning of the bound (1) inferred by the black hole formation argument depends on the interpretation of the black hole entropy. If the black hole entropy is the logarithm of the number of states of the black hole including the interior states, then we infer a “volume bound” on the entropy contained within Σ . If however, as argued above, the black hole entropy reflects only those states that can influence the exterior, then we infer only a “surface bound” on the surface states of Σ .

I do not consider the volume bound interpretation to be viable. Not only can it not be inferred from the second law with the surface interpretation of black hole entropy, but it seems contradicted by the example used in the first section of this paper: since the volume of the region interior to the surface could be arbitrarily large it could contain an arbitrarily large amount of entropy. It also suffers from a species problem, that is, the entropy inside could be arbitrarily large if the number of independent fields in nature is arbitrarily large (but see [33] for another point of view). (On the other hand, if the number of species is sufficient for an order unity violation of the bound, then a black hole would be unstable to explosive evaporation on a timescale of order the light crossing time, and so the original rationale for the bound would be lost [34].)

As an important side remark, note that the black hole formation argument suggesting the bound (1) does not apply to every closed 2-surface, since not every such surface can be made to coincide with a slice of the horizon of a black hole. Consider for instance an outer trapped surface inside a black hole. The future pointing null congruences orthogonal to this surface are converging on both sides, whereas the horizon generators are always non-converging according to the area theorem. For another example, consider the intersection of the past light cones of two spacelike related points p and q . The future pointing null congruences orthogonal to this intersection surface are converging (to p and q) on both sides. (This surface is not compact, but one can build a compact 2-surface out of pieces like this.) The restriction on surfaces is certainly necessary for the volume interpretation of the

bound (although as discussed above I do not consider this interpretation to be viable in any case), since otherwise it is easy to find surfaces with arbitrarily little area enclosing a large volume. For example, a trapped surface near the singularity of a Schwarzschild black hole can have arbitrarily small area and still bound a finite volume. For another example, one can make a spacelike surface of arbitrarily small area enclose any volume by wiggling the surface “up and down” in the timelike direction.

An interpretation of the bound (1) that is neither a volume nor a surface interpretation has been proposed by Bousso [35]. In this interpretation, S_Σ is the entropy crossing any segment of a null hypersurface, meeting Σ orthogonally, that is expanding towards Σ . The validity of this bound in a variety of contexts has been argued for in Ref. [35].

The volume bound interpretation of (1) suggests the “holographic principle” [31,36] according to which all the physics in the volume should be describable by a theory on the bounding surface Σ . The surface bound interpretation on the other hand does not have any holographic connotation. Bousso suggests that his bound motivates a holographic principle which refers to the null surface segments, but these segments do not in general span the volume. It thus seems to me that the holographic principle, while it may be a property of quantum gravity and/or of the AdS/CFT duality, is *not* logically suggested by the Bekenstein bound.

BLACK HOLE ENTROPY WITHOUT BLACK HOLES

I have argued above that black hole entropy is not determined by the number of internal states of the black hole, but rather by the number of states, associated with the presence of the horizon, that can influence the outside world. This suggests that the notion of black hole entropy should apply not just to black holes but to any causal horizon.

In fact, some approaches to computing the entropy associated with horizons do yield the result $1/4$ per Planck area of a Rindler horizon or a deSitter horizon, both of which are observer dependent horizons. For example, in a recent paper Carlip [37] finds this result from the representation theory of a conformal subgroup of the diffeomorphism group associated with any (non-degenerate) Killing horizon, and he points out that the Euclidean path integral approach also yields an entropy for deSitter horizons [38]. Also, the black hole pair creation probability is weighted by $\exp(\Delta A_{\text{accel}}/4)$ where ΔA_{accel} is the associated increase of the area of an acceleration horizon [39]. This strongly suggests a state-counting role for the entropy of acceleration horizons, an idea which is further supported by calculations relating transition amplitudes for particle creation processes to the associated change of horizon area [40]. (Ref. [41] argues that one should *not* attribute an entropy to the acceleration horizon because of its observer-dependent nature. For the reason articulated in the concluding remarks, I do not subscribe to this viewpoint.)

As a more direct way to establish the validity of horizon entropy without black

holes, I will now argue that there are general laws of horizon thermodynamics, strictly analogous to those for black holes, for a class of causal horizons which I will call “partial event horizons”. Recall that the global event horizon of an asymptotically flat spacetime is the boundary of the past of future null infinity \mathcal{I}^+ . I define similarly a partial event horizon (PEH) as the boundary $\partial I^-[p]$ of the past of a single point³ $p \in \mathcal{I}^+$. In flat spacetime a PEH is just a Rindler (acceleration) horizon, and in an asymptotically flat spacetime a PEH asymptotically approaches a Rindler horizon.

Although a PEH has cross sections with infinite area, it satisfies Hawking’s classical area theorem in the local sense that the expansion of its null generators is nowhere negative. The proof is similar to but slightly simpler than that for the event horizon since the assumption of cosmic censorship can be applied directly to rule out the possibility that a null generator leaves the PEH before reaching \mathcal{I}^+ . Thus *changes* in the area are nonnegative, so a PEH satisfies a classical “second law of horizon mechanics”.

A quasistationary region of a PEH also satisfies a “first law of horizon mechanics” that is strictly analogous to the first law of black hole mechanics $dM = (\kappa/8\pi)dA$. This law for black holes can be understood in a quasi-local fashion, called the “physical process version” in Ref. [42], which applies to variations away from a quasi-stationary configuration with approximate horizon generating Killing field χ^a . In this setting dM is interpreted as the flux $\int T_{ab}\chi^a d\Sigma^b$ of “boost energy” across the horizon or a part thereof. A generic PEH will possess many quasistationary regions, to which the physical process version of first law will apply for the same reason as for black hole horizons. (The normalization ambiguity of the boost Killing field scales both dM and κ in the same way, so the first law is independent of this ambiguity [43].

Finally, as for the generalized second law, note that Sorkin’s proposal for the origin of the generalized second law described above applies to any causal horizon, and in particular it applies to a PEH. Moreover, it seems that all gedanken experiments supporting the generalized second law for quasistationary processes involving black hole horizons would apply as well to quasistationary regions of PEH’s.

CONCLUDING REMARKS

What distinguishes a black hole horizon from a more general causal horizon is that it is universally defined with reference only to the global causal structure of the spacetime. The absence of reference to particular observers or classes of observers is thus its key distinguishing feature. In practice, however, this universality is irrelevant. For example, the universe may be spatially compact, and yet we have no reservations in applying the laws of black hole thermodynamics to approximately isolated “black holes”. It is always we who divide the system into the “outside” and the “inside”. It thus seems entirely natural that the notion of black hole

³⁾ One could of course consider the boundary of the past of any subset of \mathcal{I}^+ .

entropy extends to general causal horizons. This generalized notion of horizon entropy preserves the the formula $S = A/4\hbar G$, whose universality is understood as arising from the ultraviolet dominance of the “density of surface states”, much as the universal form of the short distance limit of quantum field correlations is understood.

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